

Math 1231 — Quiz 1 (Version B)

Limits and Continuity Sections 2.1–2.4

September 11, 2026 12 points

SOLUTIONS

Problem 1. Computing limits.

(6 points)

(a) (2 points) Circle the correct value of

$$\lim_{x \rightarrow 3} \frac{\sin^2 x + \cos^2 x}{x^2 + 2x - 6}.$$

(A) 0

(B) 1

(C) $\frac{1}{9} \leftarrow$

(D) the limit does not exist

Solution. By the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ for every x , so the numerator is the constant 1. The denominator is a polynomial, and at $x = 3$ it equals $3^2 + 2(3) - 6 = 9 \neq 0$, so the quotient law applies and we substitute directly:

$$\lim_{x \rightarrow 3} \frac{\sin^2 x + \cos^2 x}{x^2 + 2x - 6} = \frac{1}{3^2 + 2(3) - 6} = \frac{1}{9}.$$

Answer: (C).

(b) (2 points) Circle the correct value of

$$\lim_{x \rightarrow 5} \frac{x^2 - 2x - 15}{x^2 - 25}.$$

(A) 0

(B) $\frac{4}{5} \leftarrow$

(C) 1

(D) the limit does not exist

Solution. Substituting $x = 5$ gives $\frac{25-10-15}{25-25} = \frac{0}{0}$, an indeterminate form, so factor and cancel:

$$\frac{x^2 - 2x - 15}{x^2 - 25} = \frac{(x-5)(x+3)}{(x-5)(x+5)} = \frac{x+3}{x+5} \quad (x \neq 5).$$

The two functions agree everywhere near 5 except at 5 itself, so they have the same limit, and the new denominator is nonzero at 5:

$$\lim_{x \rightarrow 5} \frac{x+3}{x+5} = \frac{8}{10} = \frac{4}{5}.$$

Answer: (B).

(c) (2 points) Evaluate the limit. Show your work.

$$\lim_{x \rightarrow 9} \frac{\sqrt{x+7} - 4}{x-9}$$

Solution. Step 1. Substituting $x = 9$ gives $\frac{\sqrt{16}-4}{0} = \frac{0}{0}$, an indeterminate form. A square root calls for its conjugate $\sqrt{x+7} + 4$:

$$\frac{\sqrt{x+7} - 4}{x-9} \cdot \frac{\sqrt{x+7} + 4}{\sqrt{x+7} + 4}.$$

Step 2. Multiply out the numerator only, keeping $(x-9)$ factored below. Since $(\sqrt{x+7}-4)(\sqrt{x+7}+4) = (x+7)-16 = x-9$,

$$= \frac{x-9}{(x-9)(\sqrt{x+7}+4)}.$$

Step 3. Cancel the common factor $(x-9)$, legal because the limit ignores the single point $x=9$:

$$= \frac{1}{\sqrt{x+7}+4}.$$

Step 4. Substitute $x=9$:

$$\lim_{x \rightarrow 9} \frac{\sqrt{x+7}-4}{x-9} = \frac{1}{\sqrt{16}+4} = \frac{1}{8}.$$

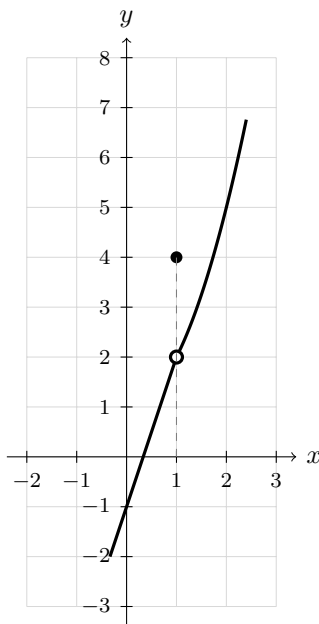
Problem 2. A piecewise function.

(6 points)

Let

$$f(x) = \begin{cases} 3x - 1, & x < 1, \\ 4, & x = 1, \\ x^2 + 1, & x > 1. \end{cases}$$

(a) (2 points) Sketch the graph of f on the axes below. Use open circles and filled dots to show clearly what happens at $x = 1$.



Solution. To the left of 1 the graph is the line $y = 3x - 1$, rising to an *open circle* at $(1, 2)$. To the right of 1 it is the parabola $y = x^2 + 1$, leaving the same *open circle* at $(1, 2)$ and curving upward. The value $f(1) = 4$ is a single *filled dot* at $(1, 4)$, floating above the hole. Both branches head for the same height 2, but the function's value there is 4.

(b) (2 points) **True or False:** $\lim_{x \rightarrow 1} f(x)$ exists. Circle one, then justify your answer using one-sided limits.

TRUE ←**FALSE**

Solution. TRUE. Compute the two one-sided limits. From the left $x < 1$, so the rule $3x - 1$ applies; from the right $x > 1$, so the rule $x^2 + 1$ applies:

$$\lim_{x \rightarrow 1^-} f(x) = 3(1) - 1 = 2, \quad \lim_{x \rightarrow 1^+} f(x) = 1^2 + 1 = 2.$$

Both one-sided limits exist and they agree, so the two-sided limit exists and $\lim_{x \rightarrow 1} f(x) = 2$. The value $f(1) = 4$ is irrelevant here: a limit describes the behavior *near* 1, never *at* 1.

(c) (2 points) **True or False:** f is continuous at $x = 1$. Circle one, then justify your answer. If it is not continuous, state which condition in the definition of continuity fails and classify the discontinuity.

TRUE**FALSE** ←

Solution. FALSE. Check the three conditions in order.

- (i) $f(1) = 4$ is defined. ✓
- (ii) $\lim_{x \rightarrow 1} f(x) = 2$ exists, by part (b). ✓
- (iii) $\lim_{x \rightarrow 1} f(x) = 2 \neq 4 = f(1)$. **Fails.**

Condition (iii) fails — the limit and the function value disagree — so f is discontinuous at $x = 1$. Because the two-sided limit exists as a real number, this is a **removable discontinuity**: redefining $f(1) = 2$ would fill the hole and make f continuous at 1.

Math 1231 — Quiz 1 (Version A)

Limits and Continuity Sections 2.1–2.4

September 11, 2026 12 points

SOLUTIONS

Problem 1. Computing limits.

(6 points)

(a) (2 points) Circle the correct value of

$$\lim_{x \rightarrow 2} \frac{\sin^2 x + \cos^2 x}{x^2 + 3x - 6}.$$

(A) 0

(B) $\frac{1}{4} \leftarrow$

(C) 1

(D) the limit does not exist

Solution. By the Pythagorean identity $\sin^2 x + \cos^2 x = 1$ for every x , so the numerator is the constant 1. The denominator is a polynomial, and at $x = 2$ it equals $2^2 + 3(2) - 6 = 4 \neq 0$, so the quotient law applies and we substitute directly:

$$\lim_{x \rightarrow 2} \frac{\sin^2 x + \cos^2 x}{x^2 + 3x - 6} = \frac{1}{2^2 + 3(2) - 6} = \frac{1}{4}.$$

Answer: **(B)**.

(b) (2 points) Circle the correct value of

$$\lim_{x \rightarrow 4} \frac{x^2 - x - 12}{x^2 - 16}.$$

(A) 0

(B) 1

(C) $\frac{7}{8} \leftarrow$

(D) the limit does not exist

Solution. Substituting $x = 4$ gives $\frac{16-4-12}{16-16} = \frac{0}{0}$, an indeterminate form, so factor and cancel:

$$\frac{x^2 - x - 12}{x^2 - 16} = \frac{(x-4)(x+3)}{(x-4)(x+4)} = \frac{x+3}{x+4} \quad (x \neq 4).$$

The two functions agree everywhere near 4 except at 4 itself, so they have the same limit, and the new denominator is nonzero at 4:

$$\lim_{x \rightarrow 4} \frac{x+3}{x+4} = \frac{7}{8}.$$

Answer: **(C)**.

(c) (2 points) Evaluate the limit. Show your work.

$$\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$$

Solution. Step 1. Substituting $x = 7$ gives $\frac{\sqrt{9}-3}{0} = \frac{0}{0}$, an indeterminate form. A square root calls for its conjugate $\sqrt{x+2} + 3$:

$$\frac{\sqrt{x+2} - 3}{x - 7} \cdot \frac{\sqrt{x+2} + 3}{\sqrt{x+2} + 3}.$$

Step 2. Multiply out the numerator only, keeping $(x-7)$ factored below. Since $(\sqrt{x+2}-3)(\sqrt{x+2}+3) = (x+2)-9 = x-7$,

$$= \frac{x-7}{(x-7)(\sqrt{x+2}+3)}.$$

Step 3. Cancel the common factor $(x-7)$, legal because the limit ignores the single point $x=7$:

$$= \frac{1}{\sqrt{x+2}+3}.$$

Step 4. Substitute $x=7$:

$$\lim_{x \rightarrow 7} \frac{\sqrt{x+2}-3}{x-7} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}.$$

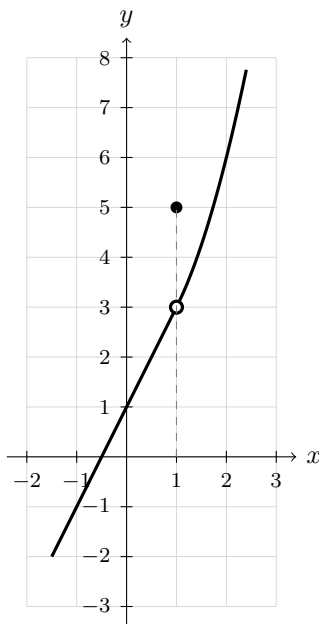
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Let

$$f(x) = \begin{cases} 2x + 1, & x < 1, \\ 5, & x = 1, \\ x^2 + 2, & x > 1. \end{cases}$$

(a) (2 points) Sketch the graph of f on the axes below. Use open circles and filled dots to show clearly what happens at $x = 1$.



Solution. To the left of 1 the graph is the line $y = 2x + 1$, rising to an *open circle* at $(1, 3)$. To the right of 1 it is the parabola $y = x^2 + 2$, leaving the same *open circle* at $(1, 3)$ and curving upward. The value $f(1) = 5$ is a single *filled dot* at $(1, 5)$, floating above the hole. Both branches head for the same height 3, but the function's value there is 5.

(b) (2 points) **True or False:** $\lim_{x \rightarrow 1} f(x)$ exists. Circle one, then justify your answer using one-sided limits.

TRUE ←**FALSE**

Solution. TRUE. Compute the two one-sided limits. From the left $x < 1$, so the rule $2x + 1$ applies; from the right $x > 1$, so the rule $x^2 + 2$ applies:

$$\lim_{x \rightarrow 1^-} f(x) = 2(1) + 1 = 3, \quad \lim_{x \rightarrow 1^+} f(x) = 1^2 + 2 = 3.$$

Both one-sided limits exist and they agree, so the two-sided limit exists and $\lim_{x \rightarrow 1} f(x) = 3$. The value $f(1) = 5$ is irrelevant here: a limit describes the behavior *near* 1, never *at* 1.

(c) (2 points) **True or False:** f is continuous at $x = 1$. Circle one, then justify your answer. If it is not continuous, state which condition in the definition of continuity fails and classify the discontinuity.

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Solution. FALSE. Check the three conditions in order.

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Condition (iii) fails — the limit and the function value disagree — so f is discontinuous at $x = 1$. Because the two-sided limit exists as a real number, this is a **removable discontinuity**: redefining $f(1) = 3$ would fill the hole and make f continuous at 1.